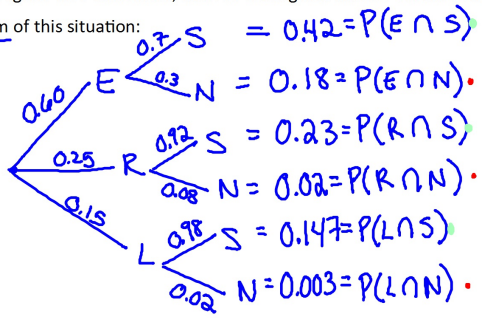
TREE DIAGRAMS

EXAMPLE: An airline offers discounted tickets to customers who buy tickets early (more than 30 days ahead of time). The company has noticed that 60% of its customers take advantage of the "early-bird" fares, while 25% purchase regular fares, and 15% are "last-minute" customers (less than 48 hours before the flight). They have also figured out that the no-show rate among "early-bird" purchasers is 30%, and only 8% among regular fare customers, and 2% among last-minute customers.

Create a tree diagram of this situation:

$P(E)=0.60$
$P(R)=0.25$
$P(L)=0.15$



So now let's answer some easy questions:

1) What is the probability of being a no-show (overall)?

$P(N) = 0.203$

2) What is the probability of being a no-show AND a last minute customer?

$P(N \cap L) = 0.003$

3) What is the probability of being a no-show AND an early-bird customer?

$P(N \cap E) = 0.18$

Now let's try some harder ones:

4. Given that you have a no-show in a certain seat, what is the probability of that person being a last-minute customer?

$$P(L|N) = \frac{P(L \cap N)}{P(N)} = \frac{0.003}{0.203} = 0.0148$$

5. Given that you have a no-show in a certain seat, what is the probability of that person being a regular-fare customer?

$$P(R|N) = \frac{P(R \cap N)}{P(N)} = \frac{0.02}{0.203} = 0.0985$$

6. Given that you have a person show up for a certain seat, what is the probability of that person being an early-bird customer

$$P(E|S) = \frac{P(E \cap S)}{P(S)} = \frac{0.42}{0.797} = 0.527$$

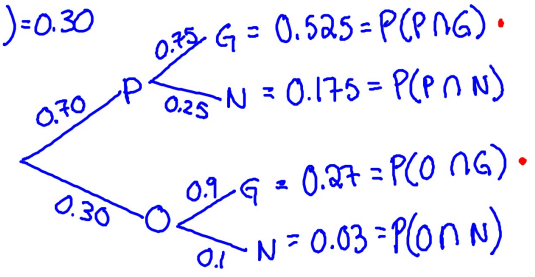
$P(B|A) = \frac{P(B \cap A)}{P(A)}$

Example #1: A college newsletter reports the following:

- 70% of incoming freshman attended public high schools
- Of those who attended public high schools, 75% eventually graduate college
- Of those who attended other high schools, 90% eventually graduate college

Create the tree diagram:

$P(Public)=0.70$
$P(other)=0.30$



Answer the following questions:

1. What is the probability that someone attends public school and graduates?

$P(P \cap G) = 0.525$

2. What is the probability that someone attends other high schools and eventually graduates?

$P(O \cap G) = 0.27$

3. What is the probability that a student graduates the college?

$P(G) = 0.795$

4. What is the probability that a student does not graduate the college?

$P(N) = 0.205$

5. What is the probability that a student is from a public school GIVEN THAT they didn't graduate?

$$P(P|N) = \frac{P(P \cap N)}{P(N)} = \frac{0.175}{0.205} = 0.854$$

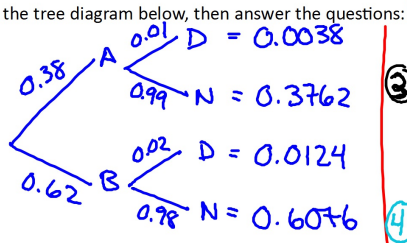
6. What is the probability that a student is from another type of school GIVEN THAT they did graduate?

$$P(O|G) = \frac{P(O \cap G)}{P(G)} = \frac{0.27}{0.795} = 0.3396$$

Example #2:

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

Make the tree diagram below, then answer the questions:



② $P(N) = 0.9838$

③ $P(B|N) = \frac{0.6076}{0.9838} = 0.6176$

④ $P(A|D) = \frac{0.0038}{0.0162} = 0.2346$

④ $P(B|D) = 0.7654$

Questions:

1- What is the probability that a book is NOT defective?

2- If a book is not defective, what's the probability that it came from company B?

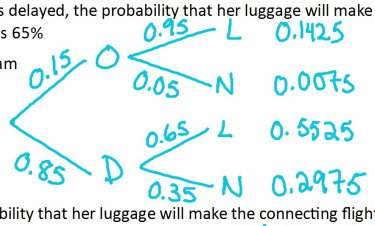
3- If we open a book and it **IS** defective, what's the probability that is from company A? Company B?

TRY THE PROBLEMS ON THE NEXT FEW PAGES

Example #3: Leah is flying home (from Boston to Denver), but she has a connecting flight in Chicago.

- The probability that Leah's first flight leaves on time is 15%. Otherwise it is delayed.
- If her first flight is on time, the probability that her luggage will make the connecting flight in Chicago is 95%
- If her first flight is delayed, the probability that her luggage will make the connecting flight in Chicago is 65%

1. Make a tree diagram



2. What is the probability that her luggage will make the connecting flight?

$P(L) = 0.695$

3. You pick up Leah at the airport and find out that her luggage did not make it (didn't make the connecting flight). What is the probability that her first flight was delayed?

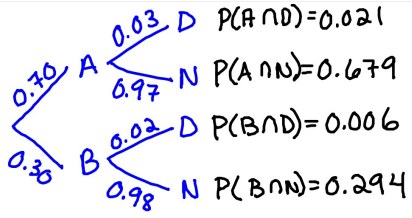
$$P(D \mid N) = \frac{0.2975}{0.305} = 0.9754$$

① $P(A) = 0.70$

$P(B) = 0.30$

$P(D \mid A) = 0.03$

$P(D \mid B) = 0.02$



ⓐ $P(D) = 0.021 + 0.006 = 0.027$

ⓑ $P(A \mid D) = \frac{P(A \cap D)}{P(D)} = \frac{0.021}{0.027} = 0.7778$

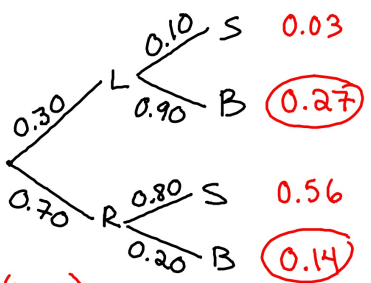
$P(B \mid D) = \frac{P(B \cap D)}{P(D)} = \frac{0.006}{0.027} = 0.2222$

② $P(L) = 0.30$

$P(R) = 0.70$

$P(S \mid R) = 0.80$

$P(B \mid L) = 0.90$



$$P(L \mid B) = \frac{P(L \cap B)}{P(B)} = \frac{0.27}{(0.27 + 0.14)} = 0.6585$$

#3

#4

complete and turn in:

p. 361 #65